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New Bucher diagrams for a class of irreversible Carnot cycles

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A Bucher diagram[1] is a graphical representation of heat flows and work in a Carnot cycle based on similar triangles (see Fig. 1). The diagram has been extended by Wallingford[2] to account qualitatively for irreversibility in a Carnot cycle (see Fig. 1) and also by the authors[3] to represent other reversible cycles in quantitative proportions. In this note we present (1) a quantitative diagram to account for irreversibility in a Carnot engine and (2) a graphical derivation of the Curzon–Ahlborn formula[4] for the efficiency of a Carnot engine at maximum power output.

In Fig. 1, an equivalent temperature T_2^* is used instead of T_2 such that the inefficiency and irreversibility of an irreversible Carnot engine are represented. It is of particular interest that the new Bucher diagram can represent quantitatively the various performances of a class of irreversible Carnot engines that are usually referred to as endoreversible Carnot engines,[5,6] because the equivalent temperature of such a class of engines can be defined clearly and accurately. Figure 2 is just a diagram that represents accurately the irreversibility of an optimal endoreversible Carnot engine.

In Fig. 2, T_1 and T_2 are, respectively, the temperatures of the cold and hot reservoirs between which the Carnot engine operates with rates of input and output heat flows, $Q_2 = Q_2/\tau$ and $q_1 = Q_1/\tau$, respectively, and power output $p = W/\tau$ where τ is the period of a cycle. Their magnitudes are represented by line segments ab, ce, and ef. The equivalent temperature T_2^* is defined[5,6] as

$$T_2^* = T_2 - q_2/k, \tag{1}$$

where $k = K_1K_2/(\sqrt{K_1} + \sqrt{K_2})^2$ and K_1 and K_2 are, respectively, the heat conductances between the working fluid and the two reservoirs at temperatures T_1 and T_2. Note that the effect from the irreversible heat conduction is represented in the diagram by the slanted line intersecting the temperature axis at T_2 and forming an angle $\varphi = \cot^{-1}(k)$ with the horizontal.

The diagram in Fig. 2 represents conservation of energy, $q_2 = q_1 + p$, i.e., ab = ce + ef. Furthermore, because the working fluid in an endoreversible Carnot engine is assumed to undergo reversible transformations and the only irreversible process is heat conduction between the reservoirs and the working fluid,[5] entropy constancy of the working fluid per cycle,

$$q_2/T_2^* = q_1/T_1, \tag{2}$$

is represented by the similar triangles in Fig. 2. The irreversible process is equivalent to a heat conduction between the reservoirs at temperatures T_2 and T_2^* in which the rate of heat flow is q_2, because the rate of entropy production of such a process at a given rate of input heat flow q_2 is

$$\sigma = \frac{q_1}{T_1} - \frac{q_2}{T_2} = q_2\left(\frac{1}{T_2^*} - \frac{1}{T_2}\right), \tag{3}$$

where Eq. (2) is used. Thus it is seen from Eq. (1) that the equivalent heat conductance of the equivalent heat conduction process is k. If T_1 is assumed to be the environmen-

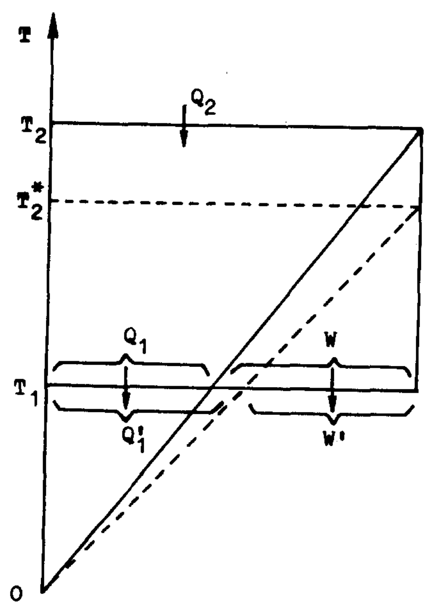

Fig. 1. Bucher diagram representing heat flows Q_2 and Q_1 and work W of a reversible Carnot engine by horizontal line segments and Wallingford's extension with modifications Q_1' and W' due to irreversibilities.

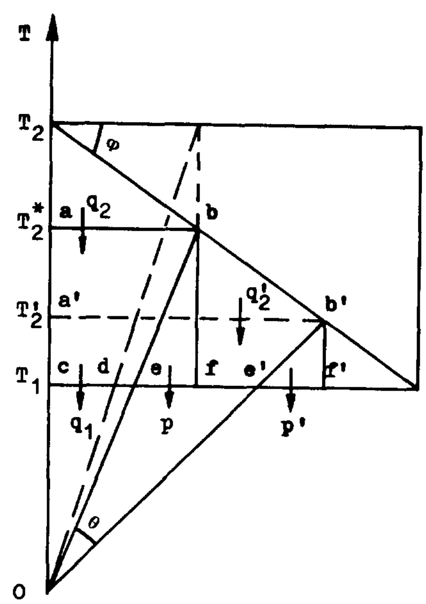

Fig. 2. New Bucher diagram for an optimal endoreversible Carnot engine.

tal temperature, line segment length de in Fig. 2 represents quantitatively the rate of loss of availability of the cycle at a given rate of input heat flow q_2.

The relation between the efficiency η and the given rate of input heat flow q_2 of the cycle,

$$\eta = \frac{p}{q_2} = 1 - \frac{q_1}{q_2} = 1 - \frac{T_1}{T_2^*} = 1 - \frac{T_1}{(T_2 - q_2/k)} \tag{4}$$

is found from the similar triangles in Fig. 2 and Eq. (1). From Eq. (4), we obtain

$$p = q_2\eta = k\eta[T_2 - T_1/(1-\eta)]. \tag{5}$$

Equation (5) is just the relation between the power output and the efficiency of an optimal endoreversible Carnot cycle.[6,7] Figure 2 or Eq. (4) shows that the efficiency η of a Carnot engine, when affected by the irreversibility of heat conduction, can attain that of a reversible Carnot engine,

$$\eta_c = 1 - T_1/T_2, \tag{6}$$

only if $q_2 = 0$. But, power output is equal to zero in such a case. Obviously, practical engines always need a certain power output. Therefore, their efficiencies cannot attain η_c.

It is easily seen from Fig. 2 or Eq. (4) that the efficiency η is a monotonic decreasing function of the rate of input heat flow q_2: When the rate of input heat flow q_2 changes from 0 to $k(T_2 - T_1)$, the efficiency η changes from η_c to 0 while the power output p changes from 0 to a maximum value $p_{\max}$ and back to 0 again. Thus the diagram in Fig. 2 can also be directly used to derive the maximum power output $p_{\max}$ and the corresponding efficiency η_m (Curzon–Ahlborn efficiency[4]) of the Carnot engine affected by the irreversibility of heat conduction. To this end we assume that when the rate of input heat flow changes from q_2 to q_2', the efficiency changes from η to η' while there is the same power output in the two cases, as shown in Fig. 2. Using the condition $p = p'$, we have

$$q_2\eta = q_2'\eta'. \tag{7}$$

From Eqs. (7) and (4), we obtain

$$T_2^* T_2' = T_1 T_2, \tag{8}$$

where T_2' is the equivalent temperature corresponding to the rate of input heat flow q_2'. It is seen from Fig. 2 that as p and p' are both increased equally, the slanted lines 0b and 0b′ will converge and the angle $\theta \to 0$. It is evident that when $\theta = 0$, $T_2^* = T_2'$, and $p = p' = p_{\max}$. Using the condition $T_2^* = T_2'$ and Eq. (8), we find from Eq. (4) that the maximum power output of the engine is given by

$$p_{\max} = k(\sqrt{T_2} - \sqrt{T_1})^2 = kT_2\eta_m^2 \tag{9}$$

and the corresponding efficiency is

$$\eta_m = 1 - \sqrt{T_1/T_2}. \tag{10}$$

Equations (9) and (10) are two important results for endoreversible Carnot engines, which have been derived recently by using the extremal condition $dp/d\eta = 0$.[6]

Figure 2 illustrates that when $p < p_{\max}$, there are two different efficiencies for a given power output, where one is larger than η_m and the other is smaller than η_m. Obviously, the larger one is the optimal efficiency.[7] Thus although the efficiency of a Carnot engine, affected only by the irreversibility of heat conduction, cannot attain η_c, the engine does not operate in the optimal condition if its efficiency is smaller than η_m. In other words, the efficiency must be laid in the optimal region between η_m and η_c such that the rate of input heat flow q_2 is not larger than $k(T_2 - \sqrt{T_1T_2})$ for such a class of engines.

Combining Eqs. (4) and (6) gives quantitatively the so-called second law efficiency[2,5] of the engine,

$$\epsilon = \frac{\text{ef}}{\text{df}} = \frac{\eta}{\eta_c} = \frac{(1 - T_1/T_2^*)}{(1 - T_1/T_2)} = \frac{1 - T_1/(T_2 - q_2/k)}{1 - T_1/T_2}, \tag{11}$$

as the ratio of the distances ef and df in Fig. 2.

Lastly, we note that the present Bucher diagram for inefficiencies and irreversibilities of a Carnot engine does not apply to irreversible Carnot refrigerators or heat pumps.[2] However, a corresponding Bucher diagram can also be constructed by using the concept of equivalent temperature introduced above.

[1]M. Bucher, "New diagram for heat flows and work in a Carnot cycle," Am. J. Phys. **54**, 850–851 (1986).

[2]J. Wallingford, "Inefficiency and irreversibility in the Bucher diagram," Am. J. Phys. **57**, 379–381 (1989).

[3]Z. Yan and J. Chen, "Modified Bucher diagram for heat flows and works in two classes of cycles," Am. J. Phys. **58**, 404–405 (1990).

[4]F. L. Curzon and B. Ahlborn, "Efficiency of a Carnot engine at maximum power output," Am. J. Phys. **43**, 22–24 (1975).

[5]M. H. Rubin, "Optimal configuration of a class of irreversible heat engine. I," Phys. Rev. A **19**, 1272–1276 (1979).

[6]L. Chen and Z. Yan, "The effect of heat-transfer law on performance of a two-heat-source endoreversible cycle," J. Chem. Phys. **90**, 3740–3743 (1989).

[7]J. Chen and Z. Yan, "Unified description of endoreversible cycles," Phys. Rev. A **39**, 4140–4147 (1989).

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A good demonstration lecture is a dramatic performance and there is no doubt that the lecturer must be at least a perfomer, if not an actor, in order to put over a demonstration lecture effectively. This may be one of the factors that leads some scientists to feel that there is something not quite respectable about such an activity.

Charles Taylor, *The Art and Science of Lecture Demonstration* (Hilger, Bristol, 1988), p. 55.